TUGAS ALJABAR LINEAR

Soal ke-1

$x_1 + 2x_2 + x_3 = 5$

$x_1 - x_2 + x_3 = 0$

$x_1 + x_2 = 0$

Untuk menemukan solusi dari persamaan di atas menggunakan algoritma *GAUSS-JORDAN* maka kita tuliskan dalam bentuk matrik sebagai berikut:

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 \\ 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} \to r1 \\ \to r2 \\ \to r3 \end{matrix}$$

Langkah berikutnya adalah proses eliminasi pada tiap baris pada matrik A sehingga membentuk *reduced row-echelon form* dari A :

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 \\ 1 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} \\ r2 \to r2 - r1 \\ r3 \to r3 - r1 \end{matrix} \begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & -3 & 0 & -5 \\ 0 & -1 & -1 & -5 \end{bmatrix}$$

$$r2 \to -\frac{1}{3} r2 \begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 1 & 0 & 5/3 \\ 0 & -1 & -1 & -5 \end{bmatrix}$$

$$\begin{matrix} r1 \to r1 - 2r2 \\ \\ r3 \to r3 + r2 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 5/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & -1 & -10/3 \end{bmatrix}$$

$$\begin{matrix} \\ \\ r3 \to -r3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 5/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 10/3 \end{bmatrix}$$

$$\begin{matrix} r1 \to r1 - r3 \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -5/3 \\ 0 & 1 & 0 & 5/3 \\ 0 & 0 & 1 & 10/3 \end{bmatrix} \begin{matrix} \to x_1 = -5/3 \\ \to x_2 = 5/3 \\ \to x_3 = 10/3 \end{matrix}$$

Jadi :

$-5/3 + 2(5/3) + 10/3 = 5 \to terbukti$

$-5/3 - 5/3 + 10/3 = 0 \to terbukti$

$-5/3 + 5/3 = 0 \to terbukti$

Soal ke-2

$$A = \begin{bmatrix} 4 & 4 & 0 & 3 \\ 4 & 1 & 0 & 1 \\ 0 & 0 & 4 & 1 \\ 4 & 14 & 3 & 6 \end{bmatrix} \begin{matrix} \rightarrow r1 \\ \rightarrow r2 \\ \rightarrow r3 \\ \rightarrow r4 \end{matrix}$$

a. Kofaktor

Dengan metode kofaktor maka determinan matrik A (det (A)) diperoleh dengan cara menghitung kofaktor berdasarkan baris pertama (r1).

$$\det(A) = \begin{vmatrix} 4 & 4 & 0 & 3 \\ 4 & 1 & 0 & 1 \\ 0 & 0 & 4 & 1 \\ 4 & 14 & 3 & 6 \end{vmatrix}$$

$$\det(A) = (a_{11} \cdot C_{11}) + (a_{12} \cdot C_{12}) + (a_{13} \cdot C_{13}) + (a_{14} \cdot C_{14})$$

$$C_{11}(A) = (-1)^{1+1} \cdot M_{11}(A) = M_{11}(A)$$

$$C_{12}(A) = (-1)^{1+2} \cdot M_{12}(A) = -M_{12}(A)$$

$$C_{13}(A) = (-1)^{1+3} \cdot M_{13}(A) = M_{13}(A)$$

$$C_{14}(A) = (-1)^{1+4} \cdot M_{14}(A) = -M_{14}(A)$$

$$M_{11}(A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 1 \\ 14 & 3 & 6 \end{bmatrix}$$

$$\det\,(M_{11}(A)) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 4 & 1 \\ 14 & 3 & 6 \end{vmatrix}$$

$$= (a_{11} \cdot C_{11}) + (a_{12} \cdot C_{12}) + (a_{13} \cdot C_{13})$$

$$= (a_{11} \cdot ((-1)^{1+1}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}))) + (a_{12} \cdot ((-1)^{1+2}(a_{21} \cdot a_{33} - a_{23} \cdot a_{31}))) + (a_{13} \cdot ((-1)^{1+3}(a_{21} \cdot a_{32} - a_{22} \cdot a_{31})))$$

$$= (1 \cdot (4 \cdot 6 - 1 \cdot 3)) + (0 \cdot -(0 \cdot 6 - 1 \cdot 14)) + (1 \cdot (0 \cdot 3 - 4 \cdot 14))$$

$$= (21) + (0) - (56))$$

$$= -35$$

$$M_{12}(A) = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 4 & 1 \\ 4 & 3 & 6 \end{bmatrix}$$

$$\det\,(M_{12}(A)) = \begin{vmatrix} 4 & 0 & 1 \\ 0 & 4 & 1 \\ 4 & 3 & 6 \end{vmatrix}$$

$$= (a_{11} \cdot C_{11}) + (a_{12} \cdot C_{12}) + (a_{13} \cdot C_{13})$$

$$= (a_{11} \cdot ((-1)^{1+1}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}))) + (a_{12} \cdot ((-1)^{1+2}(a_{21} \cdot a_{33} - a_{23} \cdot a_{31}))) + (a_{13} \cdot ((-1)^{1+3}(a_{21} \cdot a_{32} - a_{22} \cdot a_{31})))$$

$= (4 \cdot (4 \cdot 6 - 1 \cdot 3)) + (0 \cdot -(0 \cdot 6 - 1 \cdot 4)) + (1 \cdot (0 \cdot 3 - 4 \cdot 4))$

$= (84) + (0) - (16))$

$= 68$

$$M_{13}(A) = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 0 & 1 \\ 4 & 14 & 6 \end{bmatrix}$$

$$\det(M_{13}(A)) = \begin{vmatrix} 4 & 1 & 1 \\ 0 & 0 & 1 \\ 4 & 14 & 6 \end{vmatrix}$$

$= (a_{11} \cdot C_{11}) + (a_{12} \cdot C_{12}) + (a_{13} \cdot C_{13})$

$= (a_{11} \cdot ((-1)^{1+1}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}))) + (a_{12} \cdot ((-1)^{1+2}(a_{21} \cdot a_{33} - a_{23} \cdot a_{31})))$
$+ (a_{13} \cdot ((-1)^{1+3}(a_{21} \cdot a_{32} - a_{22} \cdot a_{31})))$

$= \big(4 \cdot (0 \cdot 6 - 1 \cdot 14)\big) + \big(1 \cdot -(0 \cdot 6 - 1 \cdot 4)\big) + \big(1 \cdot (0 \cdot 14 - 0 \cdot 4)\big)$

$= -(56) + (4) + (0))$

$= -52$

$$M_{14}(A) = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 0 & 4 \\ 4 & 14 & 3 \end{bmatrix}$$

$$\det(M_{14}(A)) = \begin{vmatrix} 4 & 1 & 0 \\ 0 & 0 & 4 \\ 4 & 14 & 3 \end{vmatrix}$$

$= (a_{11} \cdot C_{11}) + (a_{12} \cdot C_{12}) + (a_{13} \cdot C_{13})$

$= (a_{11} \cdot ((-1)^{1+1}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}))) + (a_{12} \cdot ((-1)^{1+2}(a_{21} \cdot a_{33} - a_{23} \cdot a_{31})))$
$+ (a_{13} \cdot ((-1)^{1+3}(a_{21} \cdot a_{32} - a_{22} \cdot a_{31})))$

$= \big(4 \cdot (0 \cdot 3 - 4 \cdot 14)\big) + \big(1 \cdot -(0 \cdot 3 - 4 \cdot 4)\big) + \big(0 \cdot (0 \cdot 14 - 0 \cdot 4)\big)$

$= -(224) + (16) + (0))$

$= -208$

$C_{11}(A) = M_{11}(A) = -35$

$C_{12}(A) = -M_{12}(A) = -68$

$C_{13}(A) = M_{13}(A) = -52$

$C_{14}(A) = -M_{14}(A) = -(-208) = 208$

$\det(A) = (a_{11} \cdot C_{11}) + (a_{12} \cdot C_{12}) + (a_{13} \cdot C_{13}) + (a_{14} \cdot C_{14})$

$= (4 \cdot -35) + (4 \cdot -68) + (0 \cdot -52) + (3 \cdot 208)$

$= (-140) + (-272) + (0) + (624)$

$= 212$

b. Eliminasi Baris

Dengan metode Eliminasi Baris, maka untuk mendapatkan determinan matrik A (det (A)) dilakukan pemrosesan pada setiap baris sehingga matrik tersebut menjadi matrik segitiga.

$$\det(A) = \begin{vmatrix} 4 & 4 & 0 & 3 \\ 4 & 1 & 0 & 1 \\ 0 & 0 & 4 & 1 \\ 4 & 14 & 3 & 6 \end{vmatrix}$$

$$\begin{matrix} r2 \rightarrow r2 - r1 \\ r4 \rightarrow r4 - r1 \end{matrix} \begin{vmatrix} 4 & 4 & 0 & 3 \\ 0 & -3 & 0 & -2 \\ 0 & 0 & 4 & 1 \\ 0 & 10 & 3 & 3 \end{vmatrix}$$

$$r4 \rightarrow r4 - \left(\frac{10}{3}\right) r2 \begin{vmatrix} 4 & 4 & 0 & 3 \\ 0 & -3 & 0 & -2 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 3 & -\frac{11}{3} \end{vmatrix}$$

$$r4 \rightarrow r4 - \left(\frac{3}{4}\right) r3 \begin{vmatrix} 4 & 4 & 0 & 3 \\ 0 & -3 & 0 & -2 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & -\frac{53}{12} \end{vmatrix}$$

Karena A telah menjadi matrik segitiga maka det (A) adalah hasil perkalian elemen-elemen diagonal utamanya, jadi :

$$\det(A) = a_{11} \cdot a_{22} \cdot a_{33} \cdot a_{44}$$

$$= (\ 4 \times -3 \times 4 \times -\frac{53}{12})$$

$$= (\ 4 \times 53\ )$$

$$= 212$$